6a) Kalkulatu $f'(2)$ deribatuagot.

$$f(x)=\frac{x-1}{x+1}$$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{\frac{2+h-1}{2+h+1}-\frac{2-1}{2+1}}{h}=$$

$$\lim_{h\to 0}\frac{\frac{1+h}{3+h}-\frac{1}{3}}{h}=\lim_{h\to 0}\frac{3(1+h)-(3+h)}{3h(3+h)}=\lim_{h\to 0}\frac{\cancel{3}+3h-\cancel{3}-h}{3h(3+h)}$$

$$\lim_{h\to 0}\frac{2\cancel{h}}{3\cancel{h}(3+h)}=\lim_{h\to 0}\frac{2}{9+3h}=\frac{2}{9}$$

b) $f(x)=\sqrt{x+2}$.

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{\sqrt{2+h+2}-\sqrt{2+2}}{h}=$$

$$=\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}=\left(\frac{0}{0}\right)\overset{H}{=}\lim_{h\to 0}\frac{\frac{1}{2\sqrt{4+h}}}{1}=\lim_{h\to 0}\frac{1}{2\sqrt{4+h}}$$

$$=1/4$$

* Horrela EZ erregelak aplikatzen gogot !!

!!

* Deribatuen erregelak aplikatu barik !!

$$=\lim_{h\to 0}\frac{(\sqrt{4+h}-2)(\sqrt{4+h}+2)}{h(\sqrt{4+h}+2)}=\lim_{h\to 0}\frac{\cancel{4}+\cancel{h}-\cancel{4}}{\cancel{h}(\sqrt{4+h}+2)}=$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}=\frac{1}{4}$$

263) 7) $f(x) = x + \frac{1}{x}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x+h+\frac{1}{x+h} - \left(x + \frac{1}{x}\right)}{h} =$$

$$= \lim_{h \to 0} \frac{h + \frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{h \cdot x(x+h) + \cancel{x} - (\cancel{x}+h)}{x \cdot (x+h) \cdot h} =$$

$$= \lim_{h \to 0} \frac{\cancel{h}(x^2 + xh - 1)}{\cancel{h}\, x\, (x+h)} = \frac{x^2-1}{x^2} = \boxed{1 - \frac{1}{x^2}}$$

b) $f(x) = \sqrt{x^2+1}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h} = \left(\frac{0}{0}\right) \text{ IND.}$$

$$\lim_{h \to 0} \frac{\left(\sqrt{(x+h)^2+1} - \sqrt{x^2+1}\right)\left(\sqrt{} + \sqrt{}\right)}{h\left(\sqrt{} + \sqrt{}\right)} =$$

$$= \lim_{h \to 0} \frac{(x+h)^2+1 - (x^2+1)}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)} =$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{1} - \cancel{x^2} - \cancel{1}}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)} =$$

$$= \lim_{h \to 0} \frac{2x+h}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}} = \frac{2x}{\sqrt{x^2+1} + \sqrt{x^2+1}} =$$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} = \boxed{\frac{x}{\sqrt{x^2+1}}}$$

243) 4) $x_0=3$. DERIBAGARRITASUNA

$$f(x)=\begin{cases} x^2-3x & x\leq 3 \\ 3x-9 & x>3 \end{cases}$$

$f_1(x)=x^2-3x$ $\quad Dom f=\mathbb{R} \rightarrow$ jarraia $x\leq 3$

$f_2(x)=3x-9$ $\quad Dom f=\mathbb{R} \rightarrow$ jarraia $x>3$

Aztertzeko jarraitasuna $x=3$ puntuan:

1.) $f(3)=3^2-3\cdot 3=0$

2.) Limitea aztertzeko

$$\left.\begin{array}{l} \lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^-}(x^2-3x)=0 \\ \lim\limits_{x\to 3^+} f(x)=\lim\limits_{x\to 3^+}(3x-9)=0 \end{array}\right\} \lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^+} f(x) \rightarrow \exists \lim\limits_{x\to 3} f(x).$$

3) $f(3)=\lim\limits_{x\to 3} f(x) \rightarrow$ Jarraia da $x=3$ puntuan.

Deribagarritasuna aztertzeko:

$$f'(x)=\begin{cases} 2x-3 & x\leq 3 \\ 3 & x>3 \end{cases}$$

$$\left.\begin{array}{l} f'(3^-)=\lim\limits_{x\to 3^-} f'(x)=\lim\limits_{x\to 3^-}(2x-3)=3. \\ f'(3^+)=\lim\limits_{x\to 3^+} f'(x)=\lim\limits_{x\to 3^+} 3=3 \end{array}\right\}$$

$\rightarrow$ Alboko deribatuak bat datoz beraz DERIBAGARRIA da $x=3$ puntuan eta $f'(3)=3$

243

7) $f(x)=\begin{cases} x^2-mx+5 & x\leq 0 \\ -x^2+n & x>0 \end{cases}$

m eta n DERIBAGARRIA IZATEKO

$f_1(x)=x^2-mx+5$ $Dom f=\mathbb{R}$
$f_2(x)=-x^2+n$ $Dom f=\mathbb{R}$ $\}$

- Jarraio izateko $x=0$ denean $f(0)=\lim\limits_{x\to 0} f(x)$

1) $f(0)=0^2-m\cdot 0+5=5$

2) $\left.\begin{array}{l}\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}(x^2-mx+5)=5 \\ \lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}(-x^2+n)=n\end{array}\right\}$ Albo limiteak berdinak izateko $\boxed{5=n}$

$\Downarrow$

$f(x)=\begin{cases} x^2-mx+5 & x\leq 0 \\ -x^2+5 & x>0 \end{cases}$

- Deribagarritasuna aztertzeko:

$f'(x)=\begin{cases} 2x-m & x<0 \\ -2x & x>0 \end{cases}$

Deribagarria izateko albo deribatuak berdinak izan behar dira

$\left.\begin{array}{l}f'(0^-)=\lim\limits_{x\to 0^-}(2x-m)=-m \\ f'(0^+)=\lim\limits_{x\to 0^+}(-2x)=0\end{array}\right\}$ $-m=0$ $\boxed{m=0}$

Deribagarria izateko $\boxed{m=0 \text{ eta } n=5}$

A5

264) **39** $f(x) = \begin{cases} \ln(x-1) & x<2 \\ 3x-6 & x \geq 2 \end{cases}$

$f_1(x) = \ln(x-1)$ $\text{Dom} f = (1, +\infty)$ Jarraia $(0, 2)$.

$f_2(x) = 3x-6$ $\text{Dom} f = \mathbb{R}$ Jarraia $(2, +\infty)$.

- Jarraitasuna aztertuz: $x=2$

1) $f(2) = 3\cdot 2 - 6 = 0$

2) $\lim_{x\to 2} f(x) = \begin{cases} \lim_{x\to 2^-} \ln(x-1) = 0 \\ \lim_{x\to 2^+} (3x-6) = 0 \end{cases}$ $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ $\exists \lim_{x\to 2} f(x) = \underline{0}$

3) $f(2) = \lim_{x\to 2} f(x) = 0 \longrightarrow$ Jarraia da $x=2$ denean

- Deribagarritasuna:

$f'(x) \begin{cases} \frac{1}{x-1} & x<2 \\ 3 & x \geq 2. \end{cases}$

$f'(2^-) = \lim_{x\to 2^-} \frac{1}{x-1} = 1$

$f'(2^+) = \lim_{x\to 2^+} 3 = 3.$

Albo deribatuak desberdinak dirauet $\rightarrow$ EZ DA DERIBAGARRIA $x=2$ DENEAN.

PUNTU ANGELUDUNA DA

A6

42/ - Kalkulatu m eta n DERIBAGARRIA izateko $\mathbb{R}$.

$$f(x) = \begin{cases} x^2-5x+m & x \leq 1 \\ -x^2+nx & x>1 \end{cases}$$

- Non $f'(x)=0$?

DERIBAGARRIA JARRAIA IZAN BEHAR DA LEHENENGO

· JARRAITASUNA.

$x \neq 1$ bada funtzioa jarraia da polinomioak izateagatik $Dom f = \mathbb{R}$.

1) $f(1) = 1^2 - 5\cdot 1 + m = -4+m$.

2) $$\lim_{x\to 1} f(x) \begin{cases} \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2-5x+m) = -4+m \\ \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (-x^2+nx) = -1+n \end{cases}$$

Albolimiteak berdinak izateko

$-4+m = -1+n \longrightarrow \boxed{m-n=3}$

3) $f(1) = \lim_{x\to 1} f(x) = -4+m = -1+n$.

· DERIBAGARRITASUNA

$$f'(x) = \begin{cases} 2x-5 & x<1 \\ -2x+n & x>1 \end{cases}$$

Deribagarria izateko albolimiteak berdinak izan behar dira.

$$\left.\begin{aligned} f'(1^-) &= \lim_{x\to 1^-} (2x-5) = -3 \\ f'(1^+) &= \lim_{x\to 1^+} (-2x+n) = -2+n \end{aligned}\right\} \quad -3 = -2+n \quad \boxed{n=-1}$$

→ Beraz DERIBAGARRIA IZATEKO:

$$\begin{cases} \boxed{n=-1} \\ m-n=3 \longrightarrow m = 3+(-1) \quad \boxed{m=2} \end{cases}$$

49) $f(x) = x \cdot |x| = \begin{cases} -x^2 & x<0 \\ x^2 & x \geq 0 \end{cases}$

a) $f'(x)$ b) $f''(x)$

a) $f(x)$ jarraia da $\mathbb{R}$ $f'(x) = \begin{cases} -2x & x<0 \\ 2x & x>0 \end{cases}$

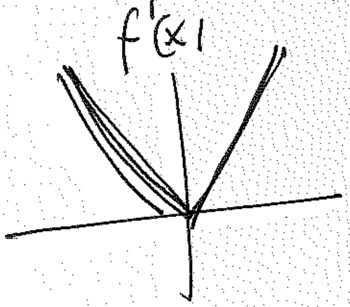
$f(0) = \lim_{x \to 0} f(x) = 0$

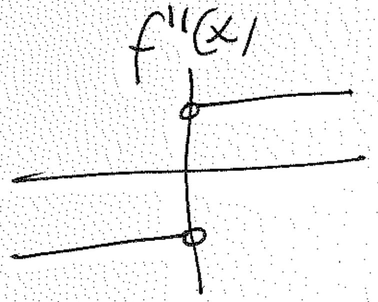
$$\left.\begin{aligned} f'(0^-) &= \lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} (-2x) = 0 \\ f'(0^+) &= \lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} (2x) = 0 \end{aligned}\right\}$$

Deribagarria da $x=0$.

$f''(x) = \begin{cases} -2 & x<0 \\ 2 & x>0 \end{cases}$

Bigarren albo deribatuak desberdinak dira. beraz $\nexists\, f''(0)$.





264.) 29 a) $x^2+y^2=9$

$2x+2yy'=0$

$y'=\frac{-\cancel{2}x}{\cancel{2}y} \longrightarrow \boxed{y'=-\frac{x}{y}}$

b) $x^2+y^2-4x-6y=-9$

$2x+2yy'-4-6y'=0.$

$y'(2y-6)+2x-4=0$

$y'=\frac{4-2x}{2y-6}=\boxed{\frac{2-x}{y-3}}$

c) $\frac{x^2}{16}+\frac{y^2}{9}=1$

$\frac{2x}{16}+\frac{2yy'}{9}=0$

$\frac{x}{8}+\frac{2yy'}{9}=0$

$\frac{9x+16yy'}{72}=0$

$\boxed{y'=-\frac{9x}{16y}}$

d) $\frac{(x-1)^2}{8}-\frac{(y+3)^2}{14}=1$

$\frac{\cancel{2}^{1}(x-1)}{\cancel{8}\,4}-\frac{\cancel{2}^{1}(y+3)y'}{\cancel{14}\,7}=0.$

$7(x-1)-4(y+3)y'=0$

$y'=\frac{7(x-1)}{4(y+3)}$

e) $x^3+y^3=-2xy$

$3x^2+3y^2y'=-2y-2xy'$

$y'(3y^2+2x)=-2y-3x^2$

$\boxed{y'=\frac{-2y-3x^2}{3y^2+2x}}$

f) $xy^2=x^2+y$

$y^2+x2yy'=2x+y'$

$y'(2xy-1)=2x-y^2$

$y'=\frac{2x-y^2}{2xy-1}$

g) $\frac{x^2}{9}-\frac{y^2}{25}=1$

$\frac{2x}{9}-\frac{2yy'}{25}=0.$

$\frac{50x-18yy'}{225}=0$

$\boxed{y'=\frac{50x}{18y}=\frac{25x}{9y}}$

h) $4x^2+4y^2+8x+3=0$

$8x+8yy'+8=0$

$y'=\frac{-8x-8}{8y}$

$\boxed{y'=\frac{-x-1}{y}}$

i/ $x^2+xy+y^2=0$

$2x+y+xy'+2yy'=0$

$y'(x+2y)+2x+y=0$

$$\boxed{y'=\frac{-2x-y}{x+2y}}$$

j) $xy-x^2-y=0$

$y+xy'-2x-y'=0$

$y'(x-1)+2x+y=0$

$$\boxed{y'=\frac{2x+y}{x-1}}$$

DERIBAZIO LOGARITMIKOA.

30

a) $y=x^{3x}$

$\ln y=\ln x^{3x}$

$\ln y=3x\cdot\ln x$

$\frac{1}{y}y'=3\cdot\ln x+\frac{1}{\cancel{x}}3\cancel{x}$

$y'=x^{3x}\cdot[\ln x^3+3)$

b) $y=x^{x+1}$

$\ln y=\ln x^{x+1}$

$\ln y=(x+1).\ln x.$

$\frac{y'}{y}=\ln x+\frac{x+1}{x}$

$y'=x^{x+1}\left[\ln x+\frac{x+1}{x}\right]$

$y'=x^{x+1}\left[\ln x+1+\frac{1}{x}\right]$

c) $y = x^{e^x}$

$\ln y = \ln x^{e^x}$

$\ln y = e^x \ln x.$

$\frac{y'}{y} = e^x \cdot \ln x + \frac{e^x}{x}$

$y' = x^{e^x} \cdot e^x \left[\ln x + \frac{1}{x}\right]$

d) $y = (\ln x)^{x+1}$

$\ln y = (x+1) \cdot \ln(\ln x)$

$\frac{y'}{y} = \ln(\ln x) + \frac{x+1}{\ln x} \frac{1}{x}$

$y' = (\ln x)^{x+1} \cdot \left[\ln(\ln x) + \frac{x+1}{x \ln x}\right]$

e) $y = \left(\frac{\operatorname{sen} x}{x}\right)^x$

$\ln y = \ln \left(\frac{\sin x}{x}\right)^x$

$\ln y = x \cdot \ln\left(\frac{\sin x}{x}\right)$

* $\frac{y'}{y} = \ln\left(\frac{\sin x}{x}\right) + \frac{\cancel{x} \cdot x}{\sin x} \cdot \frac{\cos x \cdot x - \sin x}{\cancel{x^2}}$

$\frac{y'}{y} = \ln\left(\frac{\sin x}{x}\right) + \frac{x}{\operatorname{tg} x} - 1$

$y' = \left(\frac{\sin x}{x}\right)^x \left[\ln \frac{\sin x}{x} + \frac{x}{\operatorname{tg} x} - 1\right]$

f) $y = x^{\operatorname{tg} x}$

$\ln y = \operatorname{tg} x \cdot \ln x$

$\frac{y'}{y} = (1 + \operatorname{tg}^2 x) \ln x + \frac{\operatorname{tg} x}{x}$

$y' = x^{\operatorname{tg} x} \left[(1 + \operatorname{tg}^2 x) \ln x + \frac{\operatorname{tg} x}{x}\right]$

** $\ln y = x \cdot (\ln \sin x - \ln x)$

$\ln y = x \ln(\sin x) - x \cdot \ln x$

$\frac{y'}{y} = \underbrace{\ln \sin x} + \frac{x \cdot \cos x}{\sin x} \underbrace{- \ln x} - \frac{x}{x} \Rightarrow$

$\frac{y'}{y} = \ln \frac{\sin x}{x} + \frac{x}{\operatorname{tg} x} - 1$

$y' = \left(\frac{\operatorname{sen} x}{x}\right)^x \left[\ln \frac{\sin x}{x} + \frac{x}{\operatorname{tg} x} - 1\right]$

AP

a) $y=\left(1+\frac{1}{x}\right)^x$

$\ln y = \ln\left(1+\frac{1}{x}\right)^x$ $\qquad \frac{-1/x}{x+1/x} = \frac{-1}{x+1}$

$\ln y = x\cdot\ln\left(1+\frac{1}{x}\right)$

$\frac{y'}{y} = \ln\left(1+\frac{1}{x}\right) + \frac{x\,(-1/x^2)}{1+1/x}$

$y' = \left(1+\frac{1}{x}\right)^x\left[\ln\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right]$

b) $y=(\sin x)^x$

$\ln y = \ln(\sin x)^x$

$\ln y = x\cdot\ln \sin x$

$\frac{y'}{y} = \ln(\sin x) + \frac{x}{\sin x}\cos x$

$y' = (\sin x)^x\left[\ln \sin x + \frac{x}{\operatorname{tg} x}\right]$

c) $4x^2+y^2+4x-12y-6=0$

$8x+2yy'+4-12y'=0$

$y'(2y-12) = -8x-4$

$y' = \frac{-8x-4}{2y-12}$

$\boxed{y' = \frac{-4x-2}{y-6}}$

6

d) $xy = e^x + y$

$$y + xy' = e^x + y'$$

$$y'(x-1) = e^x - y$$

$$\boxed{y' = \frac{e^x - y}{x-1}}$$

e) $\sqrt{xy} = \ln y$

$$\frac{1}{2\sqrt{xy}} \cdot (y + xy') = \frac{y'}{y}$$

$$\frac{y}{2\sqrt{xy}} + \frac{xy'}{2\sqrt{xy}} = \frac{y'}{y}$$

$$y'\left(\frac{x}{2\sqrt{xy}} - \frac{1}{y}\right) = \frac{-y}{2\sqrt{xy}}$$

$$y'\left(\frac{xy - 2\sqrt{xy}}{2\sqrt{xy} \cdot y}\right) = \frac{-y}{2\sqrt{xy}}$$

$$y' = \frac{\cancel{2\sqrt{xy}} \cdot y\,(-y)}{\cancel{2\sqrt{xy}}\,(xy - 2\sqrt{xy})} = \frac{-y^2}{xy - 2\sqrt{xy}}$$

$$\boxed{y' = \frac{y^2}{2\sqrt{xy} - xy}}$$

f) $x^2-y^2+3xy+5=0.$

$$2x-2yy'+3xy'+3y=0$$

$$y'(-2y+3x)=-2x-3y$$

$$\boxed{y'=\frac{2x+3y}{2y-3x}}$$

KATEAREN ERREGELAREKIN

32 $y=\left(\frac{x^2+1}{x}\right)^3 \rightarrow y'=3\left(\frac{x^2+1}{x}\right)^2\cdot\frac{2x\cdot x-(x^2+1)}{x^2}$

$$=3\left(\frac{x^2+1}{x}\right)^2\left(1-\frac{1}{x^2}\right)$$

$$=\frac{3(x^2+1)^2(x^2-1)}{x^4}$$

DERIBAZIO LOGARITMIKOAREKIN

$$\ln y=3\cdot\ln\left(\frac{x^2+1}{x}\right)$$

$$\ln y=3\cdot\{\ln(x^2+1)-\ln x\}$$

$$\frac{y'}{y}=3\cdot\left(\frac{1}{x^2+1}\cdot 2x-\frac{1}{x}\right)$$

$$y'=3\left(\frac{x^2+1}{x}\right)^3\frac{2x^2-x^2-1}{x(x^2+1)}=3\frac{(x^2+1)^{\cancel{3}\,2}}{x^3}\cdot\frac{x^2-1}{x(x^2+1)}=$$

$$y'=\frac{3(x^2+1)^2(x^2-1)}{x^4}$$

b) $y=(\sin x)^x$

$$\ln y=\ln(\sin x)^x$$

$$\ln y=x\cdot\ln\sin x$$

$$\frac{y'}{y}=\ln(\sin x)+\frac{x}{\sin x}\cos x$$

$$y'=(\sin x)^x\cdot\left[\ln\sin x+\frac{x}{\operatorname{tg}x}\right]$$

c) $4x^2+y^2+4x-12y-6=0$

$$8x+2yy'+4-12y'=0$$

$$2yy'-12y'=-8x-4$$

$$y'(2y-12)=-8x-4$$

$$y'=\frac{-8x-4}{2y-12} \qquad \boxed{y'=\frac{-4x-2}{y-6}}$$

d) $xy=e^x+y$

$$y+xy'=e^x+y'$$

$$xy'-y'=e^x-y$$

$$y'(x-1)=e^x-y$$

$$\boxed{y'=\frac{e^x-y}{x-1}}$$

f) $x^2-y^2+3xy+5=0$

$$2x-2yy'+3xy'+3y=0$$

$$-2yy'+3xy'=-2x-3y$$

$$y'(3x-2y)=-2x-3y$$

$$\boxed{y'=\frac{-2x-3y}{3x-2y}}$$